

Introduction to the Cohomological Theory of Dynamical Systems

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Abstract

Many questions about the dynamical properties of a system can be studied analyzing different *cocycles* over it. In general, a main problem consists in determining if a given *cocycle* is (or not) *cohomologous* to a *coboundary*. For that, it is essential to study the resolvability of *cohomological equations* associated to the system.

In this course we shall introduce all these concepts, as well as some *cohomological theories* used in Differentiable Dynamics. We will mainly concentrate our attention in classical systems, *i.e.* whose dynamics is given by a smooth \mathbb{Z} or \mathbb{R} -action. We shall also discuss some difficulties that usually appear when we are trying to solve a cohomological equation, and this will lead us to find the necessity of extending the “classical” Ergodic Theory (which just deals with invariant measures) to understand the invariant higher order Schwartz distributions of a system.

Finally, we shall discuss the problem of classification of smooth dynamical systems with “small” first cohomology space, namely *cohomology-free* or *cohomologically rigid* systems, paying especial attention to the Katok Conjecture.